

Biology:

Reproduction:

$$\frac{dP(t)}{dt} \sim a \cdot P(t)$$

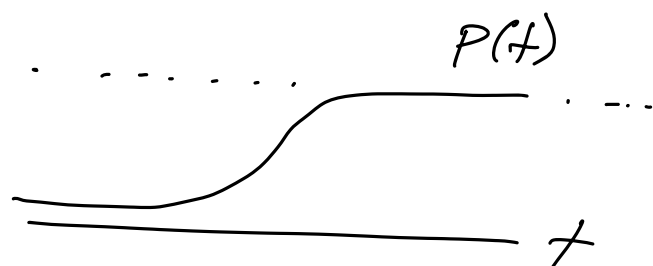
(Logistic Differential equation)

$$P(t) \sim \frac{1}{1+e^{-t}}$$

Food:

$$\frac{dP}{dt} \sim -b P(t)^2$$

$$\frac{dP}{dt} = P(t) \cdot (a - b P(t))$$



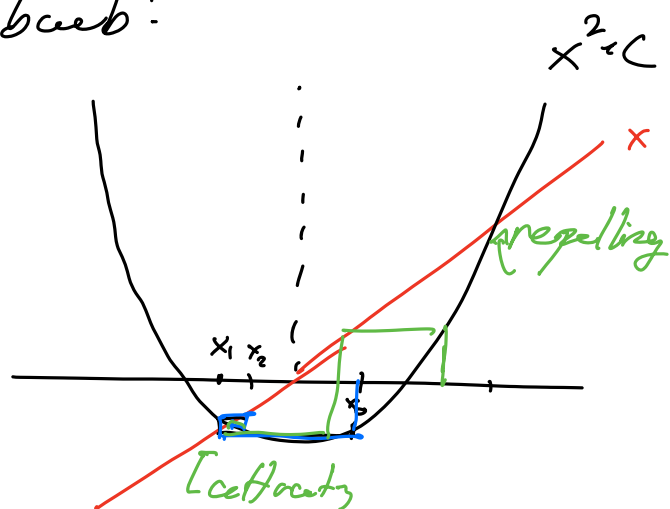
Perinchi reproduction:

(Logistic map) $y_{u+1} = y_u(a - b y_u)$

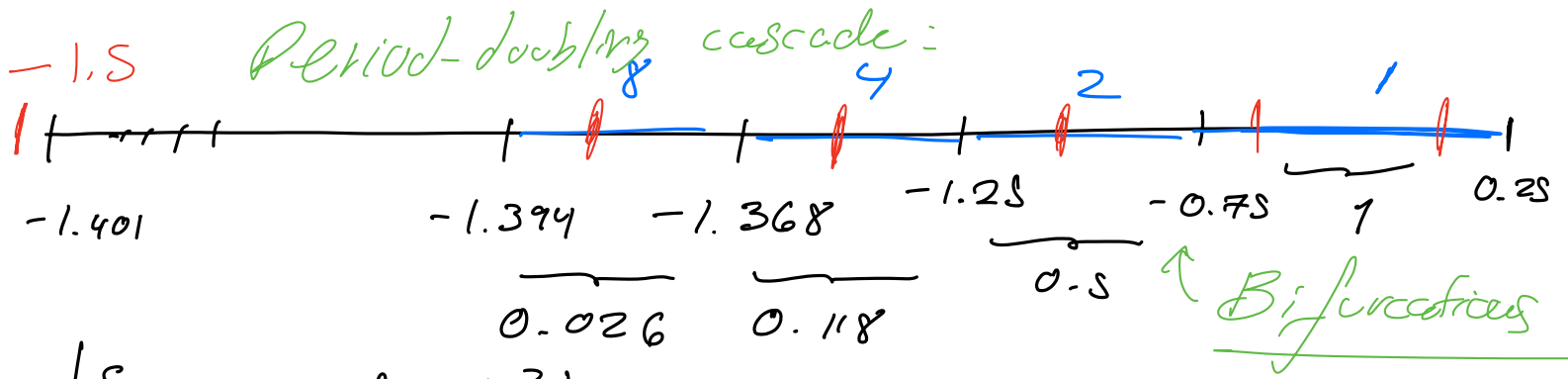
$$\leadsto x_{u+1} = x_u^2 + c = f_c(x_u)$$

Q: Fixed points, periodic cycles?

Cobweb:



Converges to an attracting fixed point



$$\frac{|\{c: \text{period } 2^n\}|}{|\{c: \text{period } 2^{n+1}\}|} \rightarrow \delta = 4.669201609 \dots$$

Feigenbaum's constant

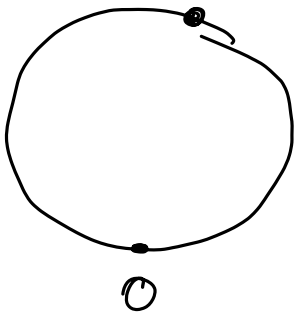
Q (Milnor): What happens for $c = -1.5$?

Windows of order: $c = -1.755$, period 3

Closely $\rightarrow$ close

Complex dynamics: $f(z) = \frac{p(z)}{q(z)} \in \mathbb{C}(z)$

$$\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\} \quad z, f(z), f^2(z), \dots$$



$$\hat{\mathbb{C}} = \underbrace{J(f)}_{\text{Julia set}} \sqcup \underbrace{F(f)}_{\text{Fatou set}}$$

$$= \{f \text{ is chaotic}\} = \{f \text{ is regular}\}$$

Ex: $f(z) = z^2$,

$$|z| < 1, \quad f^n(z) \rightarrow 0; \quad |z| > 1: \quad f^n(z) \rightarrow \infty$$

$$|z| = 1: \quad f(e^{i\theta}) = e^{2i\theta}, \quad \text{doubling}$$

$$J(f) = \{ |z| = 1 \}$$

Using IT: $c = 0, c = 0.1, c = 0.25,$
 grad deep sea $c = 0.26$ collapse

$c = -0.5, -0.75, -1$ Show
 Bifurcation two-cycle poster!

Mandelbrot set: $M = \{c \mid J(f_c) \text{ is connected}\}$
 + Show M in IT, then the correspondence

Period-stability cascade + number of
 cycles: Renormalization

Outline:

0. Review of complex analysis, Riemann surfaces,
 hyperbolic geometry

1. Fatou and Julia set:

Periodic points:

$$f^n(z) = z, \quad \lambda = (f^n)'(z) \text{ multiplier}$$

fatou julia rabbit

$\begin{cases} |\lambda| < 1: \text{attracting} & F(f) \text{ create basin of attraction in } F(f) \text{ asymptotic} \\ |\lambda| > 1: \text{repelling} & J(f), \text{ create self-similarity} \end{cases}$

$\lambda = e^{2\pi i p/q}$ parabolic: $J(f)$ rotating flowers in $F(f)$

$\lambda = e^{2\pi i t}$, $t \notin \mathbb{Q}$: depends on number theory!
 (project)

• Classification of Fatou components:

Thm. (Sullivan) Every F component is a preimage of one of the following:

- 1) an attracting basin
 - 2) a parabolic petal
 - 3) an irrationally rotating disk
 - 4) an irrationally rotating annulus.
- } number theory

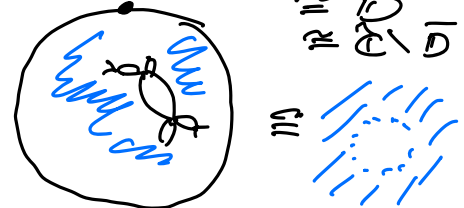
Modeling Julia sets:

Thm. (Riemann Mapping) If $U \neq \mathbb{C}, \hat{\mathbb{C}}$ is simply connected, then U is biholomorphic to $\mathbb{D}$.

• $f(z) = z^2 + c$, $K(f) = \{z \mid \{f^n(z)\} \text{ is bounded}\}$
"filled Julia set" $= \partial J(f)$

• Suppose $K(f)$ is simply connected. Then $\hat{\mathbb{C}} \setminus K(f)$ is simply connected. $\Rightarrow \hat{\mathbb{C}} \setminus K(f) \cong \mathbb{D} \setminus \overline{\mathbb{D}} \cong \mathbb{D} \setminus \overline{\mathbb{D}}$

• $J = \partial(\hat{\mathbb{C}} \setminus K)$, can we get to the boundary?



Thm. (Carathéodory - Torhorst) Riemann map $f: \mathbb{D} \rightarrow U$ extends to a cont. $\bar{f}: \overline{\mathbb{D}} \rightarrow \bar{U}$, iff ∂U is locally connected.
 $\bigcup_i S_i \rightarrow \partial U$

if so, then $J = \partial U \cong S^1 / \sim_{\bar{f}}$. Creates equiv. relation (Böttcher)

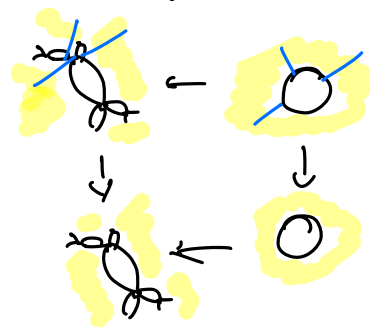
the fact, there is a special map:

$$\hat{\mathbb{C}} \setminus K(f) \xleftarrow{\sim} \hat{\mathbb{C}} \setminus \overline{\mathbb{D}}$$

$$f \downarrow$$

$$\hat{\mathbb{C}} \setminus K(f) \xleftarrow{\sim} \hat{\mathbb{C}} \setminus \overline{\mathbb{D}}$$

$$\downarrow \sim_{\bar{f}}$$



if it splits:

$$\overline{\hat{\mathbb{C}} \setminus K(f)} \xleftarrow{\bar{f}} \mathbb{C} \setminus \mathbb{D}$$

$$f \downarrow$$

$$\overline{\hat{\mathbb{C}} \setminus K(f)} \xleftarrow{\bar{f}} \mathbb{C} \setminus \mathbb{D}$$

$$J(f) \xleftarrow{\bar{f}} S^1$$

$$f \downarrow$$

$$J(f) \xleftarrow{\bar{f}} S^1$$

• Show rabbit lamination

• To find out the extension, we follow optimal rays.

Thm. Rays with distinct centers always land, even if $K(f)$ is not simply connected.

• The relation $\sim_{\bar{f}}$ is needed in the lamination

• We will show how to describe this lamination combinatorially. $\rightarrow$ pinched model

2. Mandelbrot set.

Show page 14

$$\Phi(c) = \gamma_c(c) \leftarrow \text{Unif of } J(\mathbb{Z}^2 + c)$$

- Prove that each of the components corresponds to a constant type page 15
- Give a constant model for M page 16
Conj. (MLC) M is locally connected
- Go over syllabus